CENF-Common European Numeracy Framework

Common European Numeracy Framework

Aspects and Levels

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Common European Numeracy Framework – Aspects and Levels
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Inhoud

Introduction

Numeracy, in combination with literacy and problem solving, is a key factor for adequate participation in our current information-drenched society. Although it seems obvious that citizens in the European Union have sufficient access to this, the results of the PIAAC survey (OECD, 2013) show that this is apparently not the case for about a quarter of the European adult population because of low achieved results in these subjects (OECD, 20013). The results also show that these citizens are at risk for participating in the labour market (OECD, 2016).

The European Union strives for a worldwide top position in technological developments (……). This requires sufficient higher educated European citizens to be part of this process and more mobility across the European countries. Qualitative education in compulsory school according to the 21st century skills is the basis for this and, due to the technological developments, lifelong learning is a necessity to keep up with a continuously changing society. Numeracy is in these developments a particular key to the labour market.
To establish a smooth flow of citizens within the European Union equal opportunities for education and entering the labour market must be created for all citizens. To achieve this situation the international European Qualification Framework (EQF) was developed to improve transparency, comparability and understanding of qualifications held by European individuals (European Commission, Brussels, 2016).
Ref. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52016DC0383 (visited 2021-03-19)

Part of this process was the development of a Common European Framework of Reference of languages (….. CEFR, 2006). The aim of this framework is to ensure that language levels can be internationally compared and described in a consistent way. Till today there is no system for comparing numeracy levels. With the Common European Numeracy Framework (CENF) we intend to fill the gap for numeracy.

The aim of the CENF is to create a common basis for numeracy levels across the European Union as a reference for optimal participating in society, education and the labour market across European countries. The focus in this framework is numeracy, but this cannot been seen separately from literacy and problem solving. Numeracy is a complex domain and relies heavily on sufficient literacy skills for analysing and understanding numbers in texts, and also on problem solving skills for managing situations where numbers play a role. These three components go hand-in-hand and together they are closely intertwined with digital literacy. The CENF has not been created as a new phenomenon but has been built on the numeracy frameworks of the ALL and PIAAC surveys, in order to establish recognizable connections for future international assessment. This becomes clear in the description of levels in numeracy. There is also a parallel with the CEFR in the way numeracy levels are visualized. Both were done in order to create a clear position for the CENF within the existing means for a European lifelong learning platform.

The CENF offers a numeracy framework with a level indication, a description of numeracy components, numerate behaviour and a set of example modules for professional development of teachers in non-formal adult education. During the development process of CENF a literature study

on numeracy was also done, as well as a study into numeracy activities in European countries. Summaries of both studies are published on this website.

A brief history of numeracy activities

A first signal of innumeracy came from the Cockcroft Report (1982) in the UK. The conclusion was that "functional innumeracy is far more widespread than anyone has cared to believe."(Cockcroft, 1982, pg.5). Numeracy is described as "an ability to cope confidently with the mathematical needs of adult life" (p.10).

The first international meeting on numeracy in Europe was in Marly-le-Roi in 1993, organised by the European Commission. In the same year the international association Adults Learning Mathematics was founded in 1993 in the UK. The focus is on the use of mathematics in real life situations. This was the start for international cooperation on numeracy for practitioners and researchers.

The National Numeracy Strategy (DfEE, 1999) for regular education and the Adult Numeracy Core Curriculum (The Basic Skills Agency, 2001) for adult education were developed in the UK. Together they provide a complete set for numeracy education as part of lifelong learning.

In Australia the first national curriculum for numeracy was published by the Australian Curriculum Assessment and Reporting Authority in 1996. (ACARA). Nowadays it is far more elaborated for the K-10 curriculum. (https://numeracyskills.com.au/).

Attention for numeracy in the USA started after the shocking results of the YALS (1986) and NALS, (1993) and IALS (1996) surveys. The very first numeracy framework for Adult Basic Education was published by Curry, Schmitt & Waldron (1994). Lynda Ginsburg explained in the magazine 'Adult Learning' (1997-1998) that 'Numeracy is more than Mathematics'. Gal and Tout (same magazine, reference) stated that "Real-life numeracy situations are always embedded in a life stream with real, personal meaning to the individual involved". Myrna Manly stated in the same magazine that calculators in adult education are a tool for computation, for understanding concepts, number sense and problem solving. No more ciphering and doing algorithms. This was the start for numeracy in adult education.
The national programme "Equipped for the Future" was launched in 1997. This is meant to offer "A Reform Agenda for Adult Literacy and Lifelong Learning". The focus is on lifelong learning and includes literacy and numeracy skills for adults in the 21st century.
(https://eff.clee.utk.edu/fundamentals/default.htm).

In several European countries numeracy activities were undertaken in different ways since the nineties. More co-operation across European countries started in the beginning of past decade. Several countries participated in international Grundtvig projects for exchanging information, joint activities and setting common goals for numeracy, namely: Adults Learning Mathematics Across Borders (ALMAB, 2000-2003), Mathematics in Action (MiA, 2004-2007), the European Network for Motivational Mathematics for Adults (EMMA, 2005-2007) and the In Balance project (iB-ENF, 2009-2011). The last one resulted in a first concept for a European Numeracy Framework (ENF). The need for a Common European Numeracy Framework was born. Now, one decade later, this will be realized in the CENF-project (Erasmus+, 2019-2021). With this project the basis will be led for more uniformity in describing numeracy, numerate behaviour and numeracy levels for European countries.

In the meantime the European Basic Skills Network (EBSN) was founded in 2010 as a follow-up of the EMMA project and the European Commission established the Electronic Platform for Adult Education (EPALE, since 2015) for communication across European countries. All European projects on lifelong learning are invited to publish their activities on this platform.

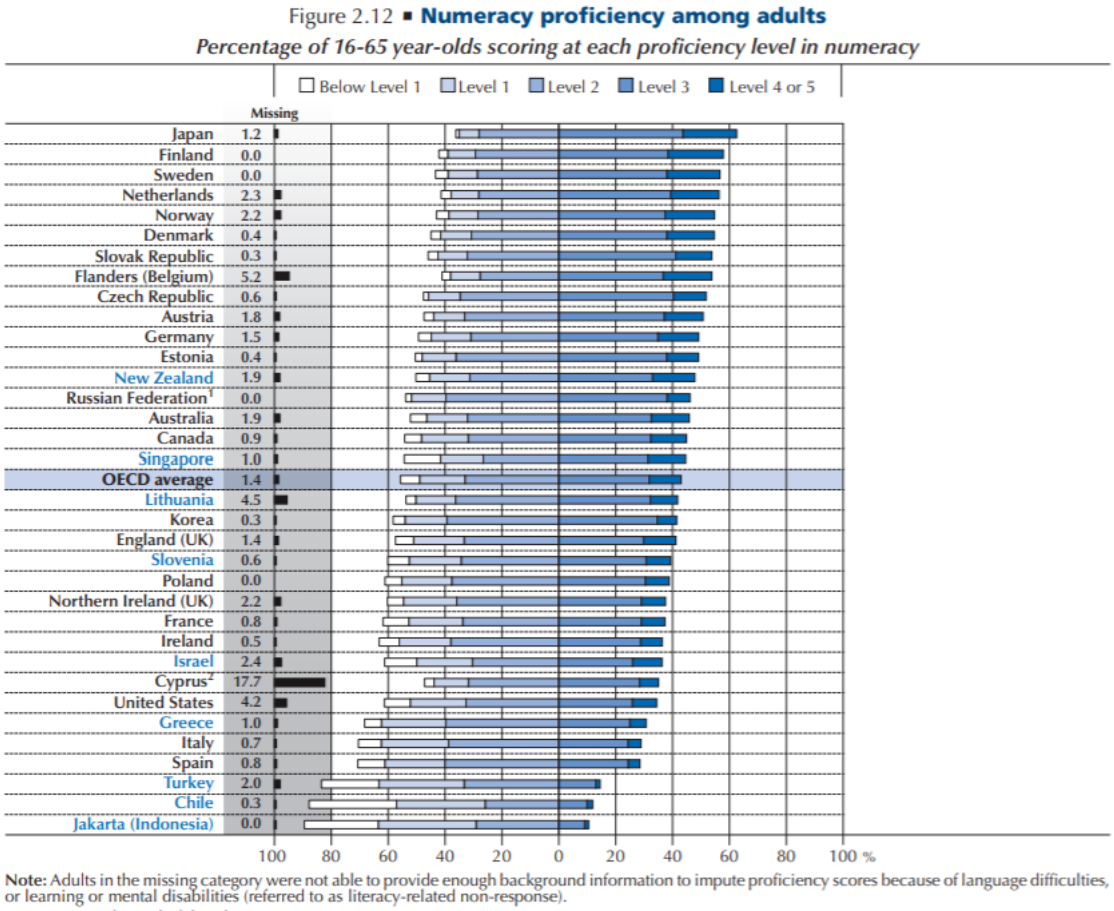
Results of international surveys

The results of the international assessments IALS (1996), ALL (2003, 2006-2008) and PIAAC (2012, 2014) have led to a sharp increase of attention to literacy and numeracy over the past twenty years. Programmes have been developed in several European countries.

The PIAAC results show that one in four adults in Europe is low-numerate (OECD, PIAAC, 2013). This means that about a quarter of the adult population is unable or insufficiently able to understand numerical information and to use numbers in all kinds of situations, such as doing calculations with units of measurement and percent.

Figure 1 shows the results of numeracy for the first cycle of PIAAC. The OECD has set the desired level for adequate participation in society at a minimum of level 3, indicated by the vertical black line. It shows how the various participating countries deviate from that.

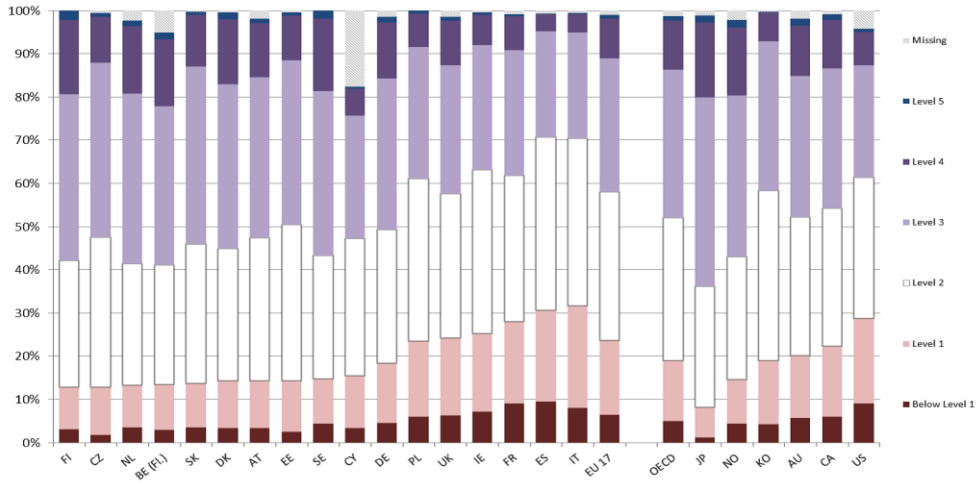
Figure 1. Numeracy proficiency among adults (PIAAC, 2013)



Source:
https://www.oecd.org/skills/piaac/Skills_Matter_Further_Results_from_the_Survey_of_Adult_Skills.pdf, (2013, p. 52)

Stewart (2015) stresses the need for numeracy in his article on the EPALE website: *One in four Europeans lack numeracy skills - enough is enough*. The results of the European countries are highlighted in figure 2.

Figure 2. *European results PIAAC Survey concerning numeracy (2013)*



Source: Stewart, J. (2015). *One in four Europeans lack numeracy skills – enough is enough.* https://ec.europa.eu/epale/en/blog/one-four-europeans-lack-numeracy-skills-enough-enough Retrieved 28-04-2015

The current gap between low-numeracy and societal participation is widening due to technological developments (…….). Low-numeracy, as well as low-literacy, can lead to social exclusion . This is a critical factor in the contemporary society. Lifelong learning is becoming increasingly necessary for everyone. This requires governments to pay more attention to numeracy (and literacy) and facilitate easily accessible opportunities for individuals to continue learning, in order to enable adults to adopt an open and adaptive attitude to new technological and social developments.

Cooperation on the reduction of low-numeracy is necessary for economic growth and sustainable development of the environment. The business community can also play an important role in this. This means that adults must be open to continuous learning and adaptation of new technological developments and the continuous changes in societal situations. Citizens must understand why this is necessary. Only then will lifelong learning become a matter of course.

The UNESCO Institute for Lifelong Learning (UIL), based in Hamburg, takes the lead in the process of global monitoring education since 2016 with their programme *Education for people and planet: Creating Sustainable futures for all.* Part of this was a draft paper for background information on numeracy by Gal (2016), in order to come to more unity in the conceptualization of numeracy. A precondition for monitoring is also a common level indication. This may make it possible to come to worldwide monitoring of adult numeracy.

In several European countries attention for the need of numeracy is increasing and activities are undertaken nowadays, but there are big differences between countries in how these activities have

been organised. It is necessary and important for the European Union to build a common basis for numeracy education to establish a good foundation for cooperation in the today's society. The ultimate basis for this is a high level of literacy and numeracy and lifelong further learning in all European countries. It is necessary to create comparable goals for numeracy and levels as is for literacy. This may improve the international flow of European citizens. With CENF a start will be made to achieve more uniformity in describing numeracy levels at a European level.

The CENF will provide such a common basis for numeracy. The development of this framework does not need to start from scratch but can be built on the examples from already existing frameworks worldwide and from ALL and PIAAC. A literature study on the concept of numeracy and a questionnaire across European countries about their policies and activities are part of this development to get more information from European countries. These are the starting points for the development of CENF.
Together with the already existing EQF and CEFR a common European basis will be created for lifelong learning and participation in society and the labour market.

Aspects of numeracy and numerate behaviour

The PIAAC study (2012-2013) defines numeracy as "*the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life".*
"A numerate adult is an individual who responds appropriately to mathematical content, information and ideas represented in various ways in order to manage situations and solve problems in a real-life context. While performance on numeracy tasks is, in part, dependent on the ability to read and understand text, numeracy involves more than applying arithmetical skills to information embedded in text." (OECD, 2012-2013. p….)

http://www.oecd.org/skills/piaac/documentation.htm, pg. 52? visited 2021-03-19

In Skills Matter: Additional Results From the Survey of Adult Skills, p.55
(ref.: PIAAC Framework 2012- Revised 28oct2013_ebook.pdf. p.33)

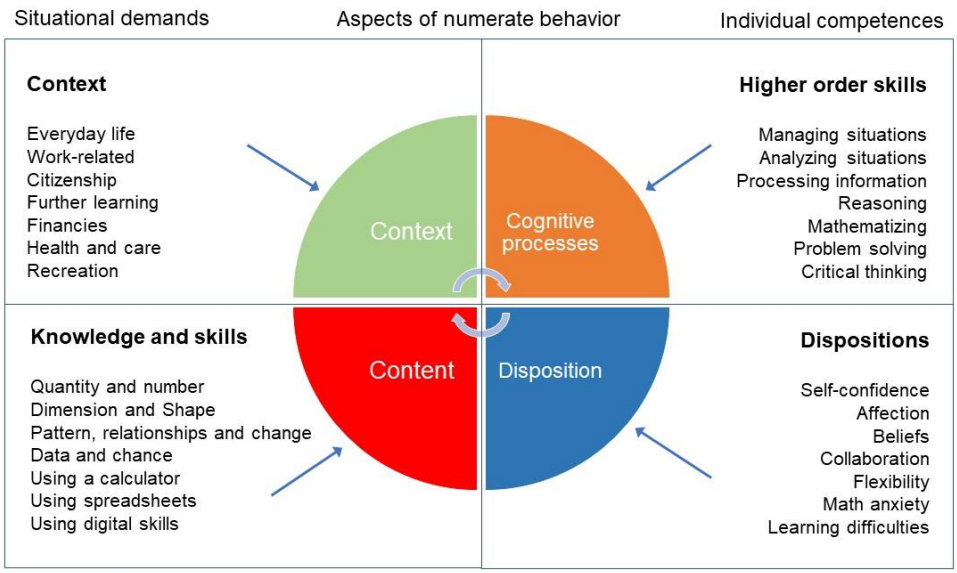
This definition is elaborated in a more explicit definition of numerate behaviour to be able to observe adults' numeracy skills, defined as "*Numerate behaviour involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways".* (ref.: PIAAC Framework 2012- Revised 28 Oct2013_ebook.pdf. p.36)

In the second cycle of PIAAC this definition is lightly changed into the combination of a definition and numerate behaviour: *Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.* (Source: Tout et al. forthcoming) (Tout, 2020). The revised numeracy definition for PIAAC Cycle 2 has two further new emphases: "*reasoning critically* with mathematical content, information and ideas *represented in multiple ways*". These changes have been done to come to agreement with the 21st century demands. (……..)

The cognitive domain "Mathematics" has a well-established and an unambiguous content description and an international "language". However, the concept of numeracy and numerate behaviour encompasses more than just substantive mathematical knowledge and skills. It is about *managing* all kinds of situations that require mathematical insight, knowledge and skills, as described in the PIAAC definition. Competencies which enable individuals to adequate actions in such situations need further elaboration. Higher order skills, dispositions with regard to mathematics, and the situation in which mathematical activities take place, play an essential role. The use of mathematical knowledge and skills (content) in daily life is always situation-specific (in context). The quality of the mathematical action depends on how the person relates himself to his or her mathematical knowledge and skills (dispositions) and the extent to which he/she can oversee and control a situation (higher-order skills). Each of the four components plays a role in one way or another in every situation (Ginsburg, Manly and Schmitt, 2006). Figure 3 shows how these aspects are mapped out in CENF.

Figure 3. Aspects of numeracy

Nieuwe afbeelding invoegen



Every adult is supposed to be able to act adequately (autonomously) in specific situations. Such does not only consist of mathematical knowledge and skills, but also of interactive and regulating knowledge and skills. The extent to which adults act adequately or efficiently depends on their familiarity with the situation in question, their dispositions to that situation and their own beliefs regarding mathematics. These, of course, do not always have to be just mathematical questions and solutions. In many cases the situation is more complex. Adults share spontaneously questions and solutions regarding such situations. It is a common social process. Such situations lead to informal learning so that the adult can act better and more efficiently in a similar next situation. This may also lead to a more conscious and critical participation in society. In this way becoming numerate is a social process (……)

In situations less familiar to an adult, it may be desirable or necessary to seek the help of experts. Decisions can be taken in consultation and with joint knowledge and skills, for example when arranging a mortgage on the purchase of a house. Such situations appeal to the adult's self-confidence with regard to his or her dispositions concerning mathematical knowledge and skills and his or her confidence in the other person. The same appeals to the other professional person. Social skills, trust and respect for each other play an important role in achieving the desired result. (……) In this way numeracy can be seen as a social construct. Adults help themselves solving problems in all kinds of individual, social and work-related situations. They are their own learners and teachers in many real life situations. (…..).

Relation to CEFR for languages

Numeracy and language go hand in hand. Numeracy competencies depend highly on language for understanding numbers in context, for understanding mathematical procedures and for mathematical reasoning. Communication concerning numeracy situations is also inherent to language. Compared to the CEFR levels there is an increase of levels in the use of mathematical language and mathematical reasoning in a native language, depending on the development of mathematical knowledge and skills of the individual adult. In a second language the development of mathematical language may be behind the level of the individual's mathematical knowledge and

skills, but may further develop depending on what mathematical language a person needs in that language. There may be a difference in the use of mathematical language for daily life and for professional use.

Relation to problem solving

The essence of numeracy is its functionality in everyday life and professional situations. Problem solving situations may be part of that and can be described as “what to do” or “what if” situations. Such situations may require mathematical thinking and communication to get at a conclusion. They may vary from easy to very complex to solve and may apply for mathematical reasoning.

Relation to digital literacy

Numeracy, Literacy and problem solving are inseparable intertwined with digital literacy nowadays. Technology offers possibilities for alternatives of doing computations by which it might seem that being numerate becomes less important. It also offers diverse opportunities for complex problem solving situations. It challenges people to continuously invent new possibilities and techniques. It appears that in particular higher educated adults profit of these developments (…….). This increases the gap between higher and lower educated adults. On the other hand, lower educated adults may also profit of these developments because it is not necessary anymore to learn and do complex computations on paper like long addition and subtraction. Though, this needs a good understanding of mathematical concepts, which procedures to use and how to do these using a calculator, mobile phones, apps or Excel. Therefore it is advised to focus numeracy activities in adult education mainly on using the latter applications. No more ciphering and learning multiplication tables.

Relation to the PIAAC Numeracy Assessment Framework

PIAAC describes four numeracy categories: numeracy for practical and personal purposes, numeracy for work purposes, numeracy for interpreting society and numeracy for further study.
In CENF a distinction is made between numeracy for practical and personal use and for social inclusion in level X, numeracy for work and society, but also for further study in level Y, and numeracy for advanced professional use and further study in level Z.

As for the numeracy domains PIAAC distinguishes quantity and number; dimension, space and shape; pattern, relationship and change; data and chance. The domains in CENF are tuned to the PIAAC domains.

Acknowledging earlier work from a similar mindset

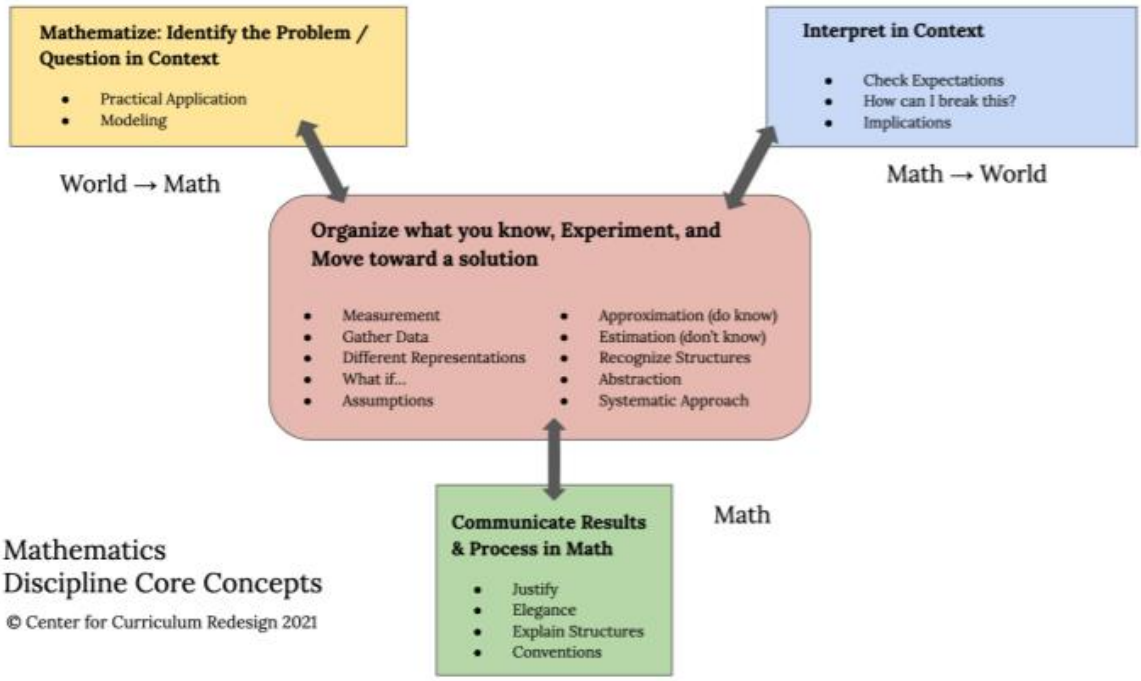
ACARA

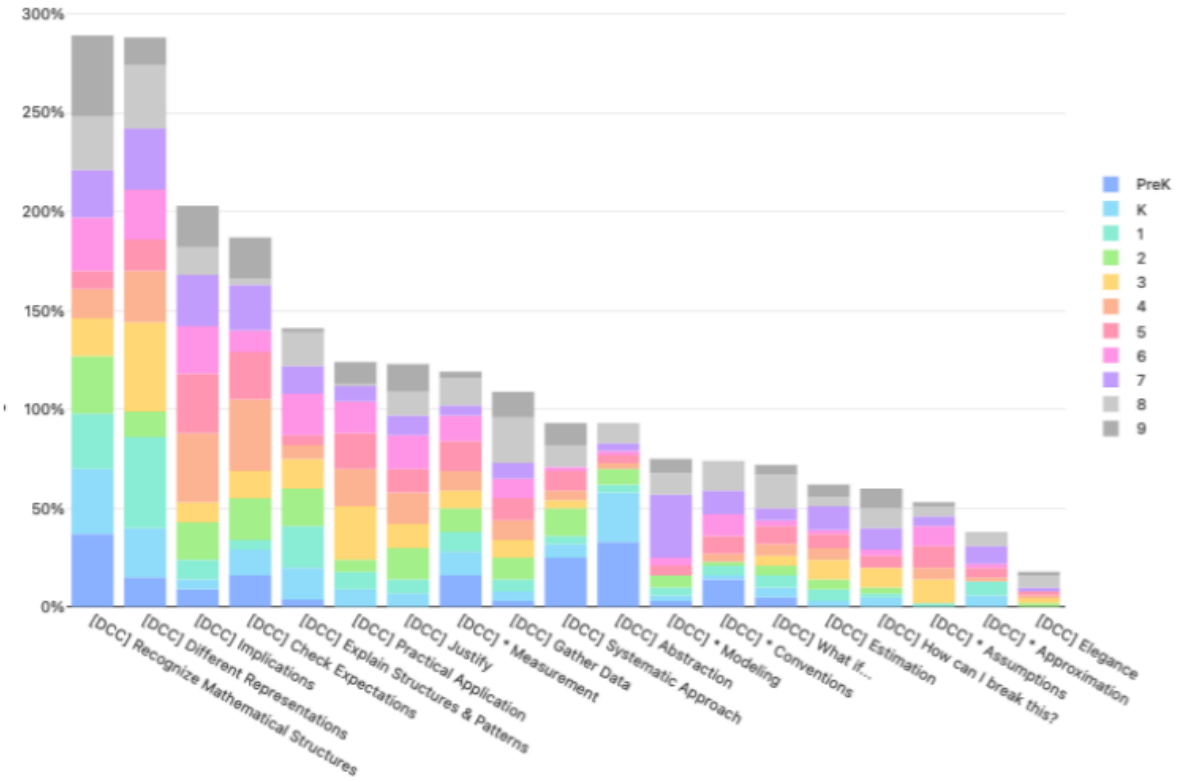
Cambridge Mathematics

Center for curriculum design

DCC = Discipline Core Concepts

These organisations all did great work in outlining the core concept descriptions including

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PreK
K
1
2
3
4
5
6
7
8
9
[DCC] Recognize Mathematical Structures
[DCC] Different Representations
[DCC] Implications
[DCC] Check Expectations
[DCC] Explain Structures & Patterns
[DCC] Practical Application
[DCC] Justify
[DCC] • Measurement
[DCC] Gather Data
[DCC] Systematic Approach
[DCC] Abstraction
[DCC] • Modeling
[DCC] • Conventions
[DCC] What if...
[DCC] Estimation
[DCC] How can I break this?
[DCC] • Assumptions
[DCC] • Approximation
[DCC] Elegance

Standing on the Shoulders of Giants

A fundamental principle underlying all of the work at CCR is taking the best of what exists in each category and use it as a starting point for further improvement. This ensures that we can remain a small and nimble team that is nevertheless able to accomplish large projects, without reinventing the wheel. We began our process of math standards design by comparing standards sets from across the world to find the ones that were most comprehensive and succinct, as well as well-aligned to our goals of concept connections, interdisciplinary connections, and competency connections.

From this analysis, we decided to start with the standards of the Australian Curriculum Authority (ACARA) as our base.48

In addition to conducting our own literature reviews, we worked closely with Cambridge Mathematics , who have been conducting in depth mapping of mathematics curriculum for years.

This helped us 49 determine the areas that most needed to be improved upon based on a synthesis of findings about the best ways to teach mathematics. We were then able to focus our efforts on the areas that needed the most change. The graph below shows the standards that A) originated with CCR, and B) originated with ACARA.

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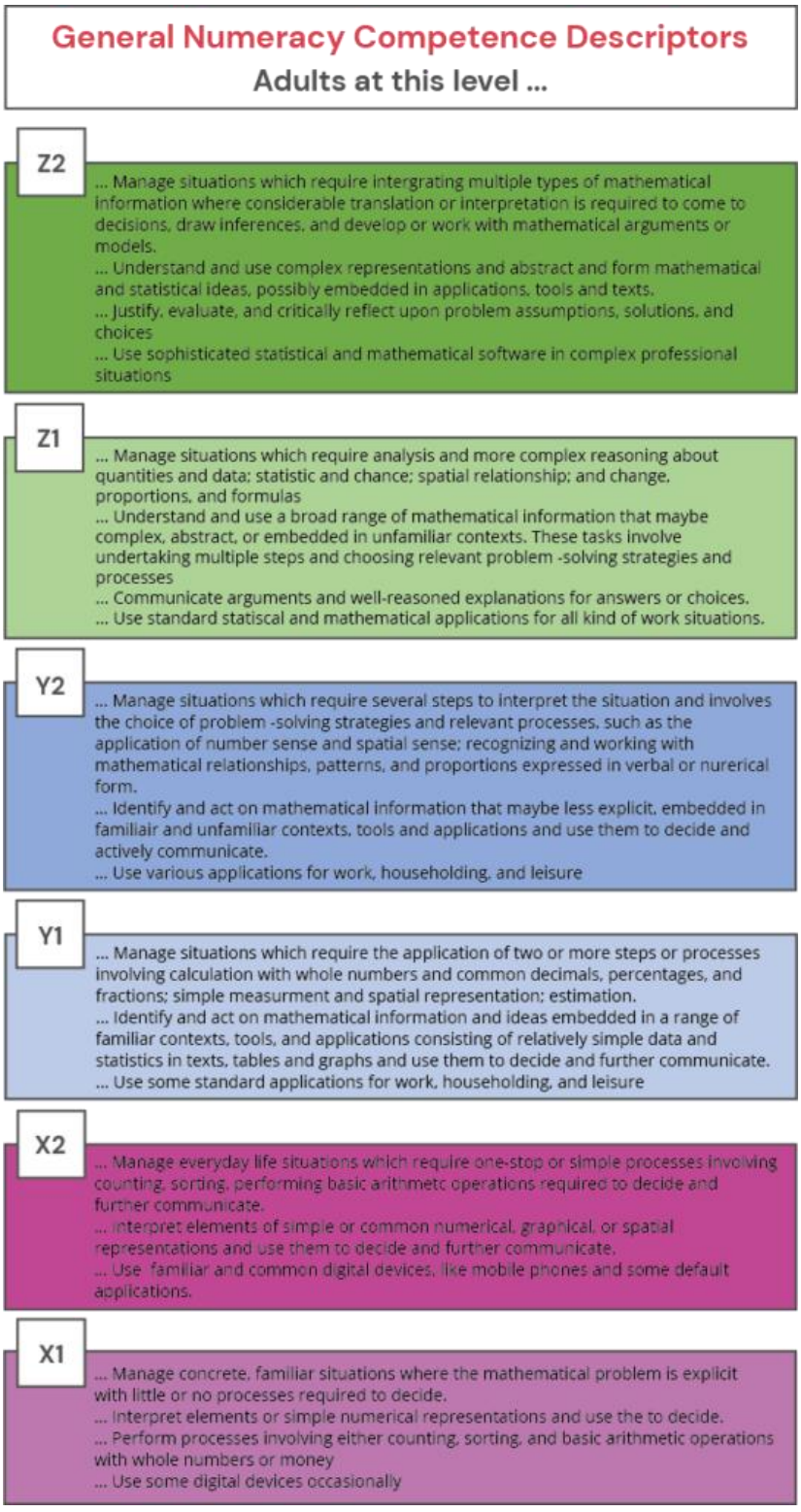
Content

Context

General Numeracy Competence Descriptors

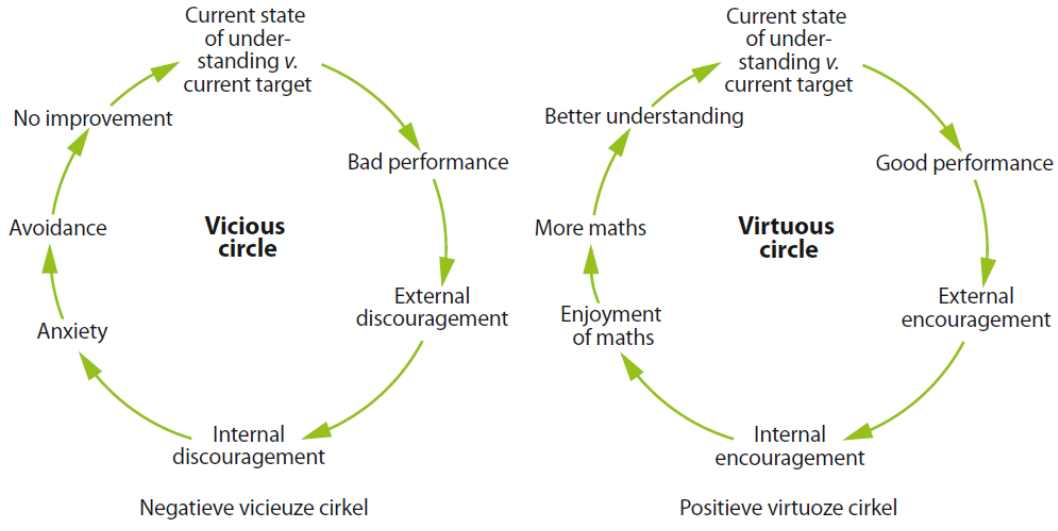


Numeracy dispositions

Dispositions that may influence numerate behaviour

Adult numeracy dispositions have been developed in the course of life, starting from childhood and compulsory school. They depend on internal capacities, beliefs and feelings concerning mathematics but have been influenced by external positive and negative experiences. It depends strongly on what and how they learned mathematics in school. The more positive experiences, like good feelings with numbers, good marks in school, passing tests, pleasure and building self-confidence when doing and using mathematics, the more positive dispositions will be developed. However, when the person meets negative experiences, like failing on mathematics tests, this may end up in negative feelings concerning mathematics and thereby also to numeracy. This is shown in Butterworth's (1999) vicious circles.

Protocol ERWD



Afbeelding 2.1 *De negatieve vicieuze cirkel en de positieve virtuoze cirkel (ontleend aan Butterworth, 1999, pp. 283-284)*

Positive experiences may lead to good dispositions for numeracy. Adults feel confident with numbers and recognize the usefulness of numeracy. On the opposite, negative experiences may develop negative feelings for numbers, less self-confidence or even math-anxiety. Disappointments may have enormous influence on numeracy capacities of the future adult.

The results of mathematics experiences during school education become visible in numeracy situations where adults have to deal with their former school experiences.

The way in which adults show their numeracy capacities in situations where they are involved in, may depend on their understanding of and fluency in mathematics.

But even when adults have acquired a good or reasonable level of mathematics fluency in combination with positive dispositions, it may depend on specific situations how they act. When they meet persons who have acquired a higher level of mathematics and more self-confidence, then they may feel a bit uncomfortable. Similar can be said about situations where adults meet other persons with less mathematical knowledge and skills. Then they may feel more comfortable in numeracy situations.

Numeracy dispositions depend on several factors like self-confidence, affection, belief, creativity, flexibility and collaboration, but also on possible experiences with learning difficulties which may lead to math anxiety.

Concerning *self-confidence,* adults may feel more or less sure of their own mathematical knowledge and skills (competencies) needed in situations that require solving a numeracy-related problem. The better adults have mastered such competencies, the more self-esteem and self-respect they may have developed.
This may influence their *flexibility, adaptivity* and *creativity* on mathematical actions when solving numeracy-related problems at work or other situations.
In many professional situations adults will have to collaborate with colleagues on projects, new products or (better) efficiency in processing systems and procedures. Such situations require *professional discussion, mathematical reasoning, flexibly exchanging* of ideas. They may also have to *create* or *adapt to* new technological possibilities. This requires *adaptivity and critical reflection* together with colleagues how to deal with that. All dispositions are inter-related and depend on the adults' positions in situations in which they have to deal with numeracy-related challenges.

Results may depend on the mathematical knowledge and skills adults acquired in former school concerning their *beliefs of usefulness* of mathematics. The more adults are convinced about usefulness, the better they may deal with numeracy activities. But adults who encountered *learning difficulties* during their schoolyears may be less lucky. They may encounter problems in diverse numeracy situations and thereby may have developed *math anxiety* or get even *blocked* when they have to deal with numbers.

Adult education, continuous learning

In adult education learners and teachers are confronted with those experiences and have to deal with it in learning environments. Teachers must be able to observe positive and less positive dispositions of their adult learners and to support them in overcoming the negative ones by offering challenging numeracy activities.

How can teachers recognize dispositions?
Numeracy fluency in combination with dispositions are the core of numerate behavior. Adults demonstrate their numerate behavior by actually doing numeracy activities, in particular in professional and learning situations.

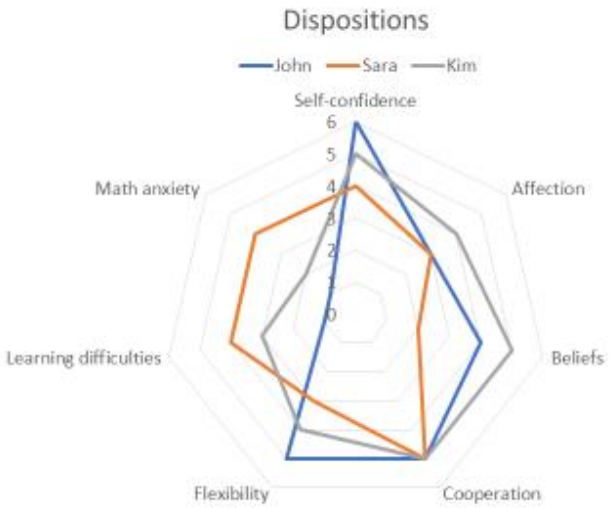
- *Self-confidence* is shown by the fluency, pace and efficiency of doing mathematical computations related to the numeracy problems to solve.
- *Flexibility, adaptivity and creativity* are shown in the way in which adults deal with new (technological) numeracy challenges.
- *Beliefs* can be observed in discussions about the usefulness of mathematics.

- *Cooperation* can be observed in situations that require professional discussion, mathematical reasoning, exchanging ideas, doing problem solving and making decisions.
- *Affection* can be observed in numeracy situations where adults show their pleasure or dislike concerning numeracy activities.
- *Learning difficulties* and *math anxiety* can be observed when adults start talking about their experiences in the past and when they hook off from numeracy activities.

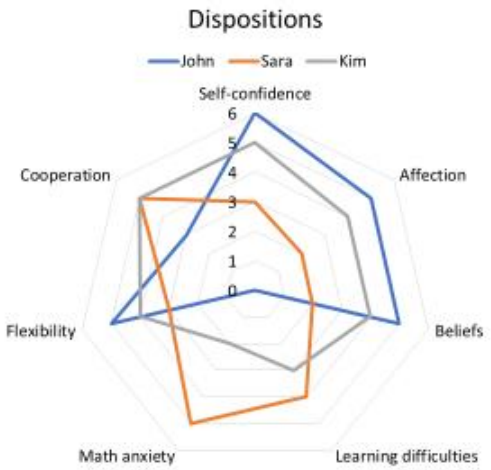
Adults on all levels of proficiency may show different levels of dispositions. In CENF we restrict these possibilities to three levels.

	1	2	3
Self-confidence	Low	Somewhat	High
Affection	I don't like math	Somewhat	I love math
Beliefs	Math is not important at all	Somewhat; depends on the situation	Math is very important
Cooperation	No cooperation	Somewhat; depends on the situation	Cooperation is important
Flexibility, adaptivity, creativity	Not flexible, adaptive, creative	Somewhat; depends on the situation	Very flexible, adaptive, creative
Learning difficulties	Severe learning difficulties	Somewhat	No learning difficulties
Math anxiety	Severe math anxiety	Somewhat	No math anxiety

Dispositions



Dispositions



Explanation

The person has acquired:

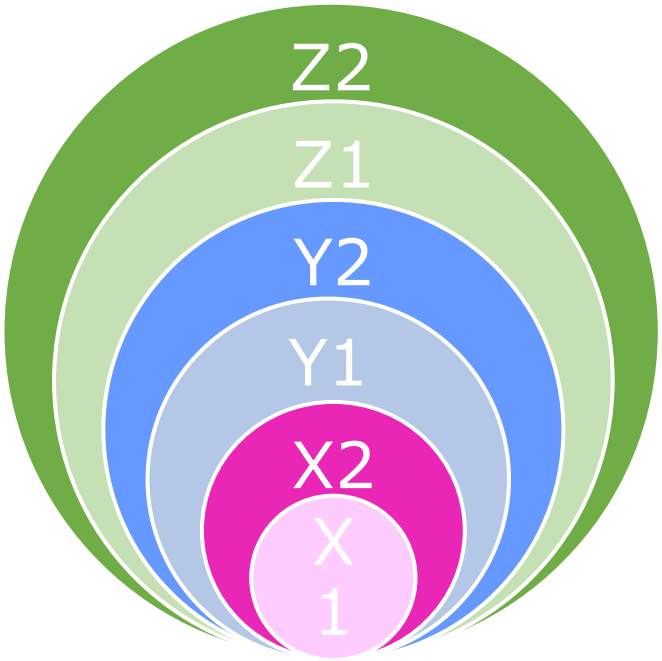
is able to…..

Can work in cooperation if…..

Levels of numeracy and numerate behaviour

To make it possible to observe adults' numerate behaviour an indication of six levels has been described in CENF. However, these levels don't have a straightforward empirical basis.
Numerate behaviour can only be observed in actual situations where adults use numbers.
In general it is rather difficult to determine levels of numerate behaviour of individual adults. Such can only be observed in real life situations where adults use mathematics. The level of numerate behaviour depend on the familiarity with and the complexity of the situation in which adults need to act. Adults can function on more levels, depending on their specific expertise in some domains as a professional and in other domains. There are no straight borders between the levels. Figure 4 shows different levels of performance in managing situations from a numeracy perspective.

Figure 4: Level indication of numerate behaviour.



Levels X1 and X2 refer to adults whose numeracy proficiency mainly takes place in familiar contexts and needs basic mathematical knowledge and skills. Level X2 describes the minimum knowledge and skills that every adult needs to be able to act independently in personal daily life. Both levels focus on social inclusion and practical work that requires simple operations.

Levels Y1 and Y2 refer to adults whose numeracy proficiency is related to their participation in society and in work situations that are not specifically mathematical of nature but may need mathematical activities. In work situations a distinction can be made between work for which lower vocational training is sufficient (Y1) and work that calls on higher vocational education (Y2). The focus in these levels is on adequate and autonomous participation in society and work situations.

Levels Z1 and Z2 refer to adults whose numeracy proficiency demands specific and specialized requirements which are set for positions that call on higher and university education. People can adapt to roles and functions in complex social or professional situations, for example as a treasurer or financial manager of an organization or company, or as a scientist or medical specialist.

Relation to CEFR for languages

In the CEFR for languages a similar distinction as for CENF is used for a broad and more holistic perspective on the aspects of literacy. The three main levels are divided in six sublevels as is in CENF: basic user for personal and family use (levels A), independent user for work, leisure and further study (levels B), and proficient user for optimal communication through texts, professional discussions and very fluently even in complex situations (levels C). These levels can be compared to the levels X, Y and Z in CENF. The focus in CENF is on the correct use of mathematical language and mathematical reasoning at the different levels, but these are not explicitly described per level.

Relation to the PIAAC levels

The levels in CENF have been described in a similar way of complexities as in PIAAC but focus on numerate behaviour. The level indication in PIAAC is ranked from <1 to level 5. The levels in CENF are ranked from 1 to 6.

Relation to EQF

These levels, along with their descriptors, make it possible to compare qualifications from different countries and institutions conform the EQF standards. The EQF is defined by eight learning outcomes-based levels. Accompanying level descriptors show how expectations of knowledge, skills, autonomy and responsibility increase as learners progress from level 1 to level 8 on the EQF scale.

(see attachment for comparison of the CENF numeracy levels to the EQF levels)

A common language and clear standards and descriptions could also lead to internationally recognized “numeracy badges”, which could be of value for citizens and employees migrating across Europe. (See for instance ebawebsite.net/badge-systems, co-funded by the European Union’s Programme Erasmus+)

Overview levels in frameworks

CENF levels	CEFR	PIAAC	EQF levels
X1	1	<1	1
X2	2	1	
Y1	3	2	2
Y2	4	3	3-4
Z1	5	4/5	5-6
Z2	6		7-8

Numeracy levels in CENF.

CENF	**General Numeracy Competence Descriptions**
levels	Adults at this level:
Z2	… manage situations which require integrating multiple types of mathematical information where considerable translation or interpretation is required to come to decisions, draw inferences, develop or work with mathematical arguments or models. … use complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts and/or graphical representation. … identify and act upon situations which require justification, evaluation and critical reflection, such as assumptions, solutions and choices. … use sophisticated statistical and mathematical software in complex professional situations.
Z1	… manage situations which require analysis and more complex reasoning about quantities and data; statistics and chance; spatial relationships; and change, proportions and formulas. … use a broad range of mathematical information that may be complex, abstract or embedded in unfamiliar situations. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes. … identify and act upon situations which require communicating arguments and well-reasoned explanations for answers or choices. … use standard statistical and mathematical applications for all kind of work situations.
Y2	… manage situations which require several steps to interpret and involve the choice of problem-solving strategies and relevant processes … use procedures that require the application of number sense and spatial sense; recognizing and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form. … identify and act upon mathematical information that may be less explicit, embedded in familiar and unfamiliar contexts, tools and applications and that apply for communication and making decisions. … use various applications for work, householding and leisure.
Y1	… manage situations which require two or more steps or processes … use procedures involving calculation with whole numbers and common decimals, percent and fractions. … identify and act upon mathematical information and ideas embedded in a range of familiar contexts, tools, and applications consisting of relatively simple data and statistics in texts, tables and graphs and that apply for communication and making decisions. … use some standard applications for householding, work and leisure.
X2	… manage everyday life situations which require one-step or simple processes … use basic mathematical procedures that require simple actions and operations with clock, calendar and standard measures. … identify and act upon common graphical and spatial representations that apply for communication and making decisions. … use familiar and common digital devices, like mobile phones and some default applications.
X1	… manage familiar situations where the mathematical information is explicit present with little or no processes required to make decisions. … use simple numerical procedures, such as counting, sorting, performing basic arithmetic operations with whole numbers or money and informal measures. … identify and act upon simple numerical and spatial representations that apply for communication and making decisions.

	… use some digital devicesPDM -

PDM

PDM - introduction

In CENF it is assumed that adults who function at the lowest levels of PIAAC, the levels X1 and X2, are the most vulnerable in society. Though it is also possible that adults at higher levels may have developed insufficient numeracy skills to function adequately in society, some of them due to mathematics difficulties, others because of poor education.
In several European countries organized courses have been offered for further learning, but it is well-known that vulnerable adults hesitate to participate in such courses due to negative school experiences in the past. For these adults more informal learning trajectories in a familiar setting may make it easier for them to participate. However, it is also well-known that, when they don't experience success in adult education they will leave the course and may get lost. Such requires qualified teachers who understand their situations and are able to encourage and motivate their participants to overcome their negative feelings for numbers and mathematics.
The focus of professional development in CENF is on teachers for adults at levels X1, X2 and Y1. It is in particular important that teachers at these levels are able to challenge and encourage adults to come to a better position in society as an outcome of numeracy (and literacy) courses. Adults at higher levels may also be low-numerate but many of them may be able to compensate or find other ways to upgrade their numeracy skills as part of further learning or professional development.

The ultimate goal of the CENF project is to have low-numerate adults on board of well-developed learning trajectories and guided by qualified teachers. The question is what kind of expertise may be expected from teachers and volunteers in order to create professional settings for adult numeracy education that is sustainable for the future. In this introduction we will analyze what knowledge and skills teachers and volunteers should be familiar with to be competent for adult numeracy education.

A brief history

In past decades initiatives were taken worldwide for improving mathematical skills of adults. One of the first well-known courses came from Paulo Freire (1970). His theory existed of learning by experiences. Starting point is the dialogue between the learners and the teacher. The teacher takes part in the discussion and creates a situation in which the learners are their own teachers. In this way learning and teaching is a mutual process in which the teacher can also be a learner. The subjects discussed are always based on real life situations that need to be managed. The main goal of Freire was working on self-awareness and social inclusion for which the learners themselves created their own learning. This way of learning and "teaching" implies respect for the learners and found its way also in literacy courses in European countries.

A first international project in Europa was *'Adults learning mathematics across borders'* (ALMAB), a Grundtvig project of Belgium, Denmark, Norway and The Netherlands (2001-2003) (Van Groenestijn (ed), 2003). This project was succeeded by the project '*Mathematics in Action*' (MiA, 2004-2007), in which the countries Denmark, Hungary, Lithuania, Netherlands, Norway, Slovenia and Spain participated. The project focused on the learning and teaching of mathematics in adult education (Van Groenestijn and Lindenskov (ed), 2007). The MiA project was succeeded by the *'European Network for Motivational Mathematics of Adults'* (EMMA, 2007-2009), of which Norway was in the lead. This network resulted into the European Basic Skills Network (EBSN), established in 2010. The EBSN encourages professional education of basic skills for adults in European countries. In addition the European Platform for Adult Learning in Europe (EPALE) was setup in 2014. This platform facilitates communication between professionals and researchers who are involved in adult education in European countries.

In parallel there was a first initiative to come to a European Numeracy Framework (ENF) that was embedded in the Grundtvig project *inBalance* (2009-2011). The Netherlands, Austria, United Kingdom, Hungary, Finland and Spain participated in this project. The result was a first setup of a European Numeracy Framework, including an internet-based program with numeracy tasks on three levels in five languages, created and trialed by teachers in the participating countries. For the validation of the tasks the levels and the complexity scheme of the ALL survey were applied (Gal et al, 2003; Tout et al, 2020). The levels were validated by Kings College in London (Van Groenestijn, Diez-Palomar, Kanes, 2011).

Learning and teaching in adult education arise five core questions that will be discussed in the next sections: principles of adult learning, teaching in adult education, teaching in adult education, what content to teach and how to teach. In section 5.7 a standard setup of PDM's is described for teacher training.

Principles of adult learning

During the Grundtvig project Mathematics in Action (MiA, 2004-2007), ways of adult learning and teaching were studied and described in the MiA handbook for teachers in adult education (Van Groenestijn & Lindenskov, ed. 2007). The participating teachers practiced a way of learning and teaching through problem solving with their participants. The findings resulted from that project may make sense for the CENF project. Teachers' comments are attached in appendix 2. In the MiA handbook it is said that

"Adults have multiple tasks to fulfill in everyday life. They have families, are parents, neighbors, citizens, customers, consumers, employers or employees, patients, members of sport clubs, volunteers in organizations, etc.. In all those situations they have their own specific roles, tasks and responsibilities which require an aggregate of integrated social, literal and mathematical competencies that they learn in the course of life. Knowledge and skills acquired in former school years are the necessary basis for this, but in general this is only a small part of what is actually needed to manage everyday life situations. " (MiA handbook, 2007, p. 22-23).

Learning in practice often means joint learning by showing and doing, based on shared experiences and responsibilities. The way in which people learn in lived-in situations takes place by communication and interaction. Learning activities are based on shared knowledge rather than individual knowledge, but shared knowledge also enhances individual knowledge. There is no teacher. Everybody can be learner and teacher. The society as a whole is in continuously further development and everybody takes part in that process (Greeno et al, 1999). In general adults learn a lot in informal situations in the course of life. There is no theory about adult learning, but some specific characteristics can be mentioned. These are described in the MiA project and copied below (Van Groenestijn, 2002, pp 184-187; Van Groenestijn & Lindenskov, 2007, pp 23-25).

1) *Adults are free to learn* (Rogers, 1969). There is no compulsory education. Adults want to learn if there is a need to learn. They want to improve specific skills or to acquire more specific knowledge to be better informed.

2) *Leaning happens in a functional situation* (Resnick, 1987). Every situation is a source for learning but is also the context in which prior acquired knowledge and skills are applied. The advantage is that new knowledge and skills are straight applicable. The disadvantage can be that knowledge and skills are situation-bound and that there is little or no transfer to similar situations in a different context.

3) *Learning in practice is characterized by learning through authentic materials.*
Whereas in school learning often takes place through text books, photos, schemes and with the help of artificial hands-on materials, in practice this can be done in the actual situation with authentic materials. For example: computing the area of a floor and determining how many tiles are necessary to cover the floor can be done with the dimensions of the actual floor and tiles and by using professional tools. Such materials may make it easier for adults to understand the mathematical situation and to analyze and solve the problem. This *learning-by-doing* leads to *knowing-for-doing* and is the basis for functional numeracy. Whereas in school students often learn *subjects* because they *should* know it, in practice subjects are learned because people *need* or *want to know* it, to be

able to do their jobs or other things. Knowledge acquired in practice is almost always functional and applicable.

4) *Every learning situation is a socio-cultural determined situation.* Saxe (1991) describes, referring to Vygotsky (1978), that social interactions are redirected by cultural and historical influences. These affect natural processes in cognitive development. In essence, learning is an *interactive and social act* in which everybody takes part. *Communication* by talking about problems which need to be solved and in what way, is an essential part of the learning process and a starting point of developing reasoning skills and problem solving strategies and skills.

5) *Learning in practice focuses on "shared cognition", rather than on "individual cognition"* (Resnick, 1987). Whereas students in school are assessed on their individual knowledge and competencies, adults may learn from and to each other in lived-in situations where they actually need each other.

6) The way in which learning in practice takes place is often via *showing, imitating, participating and applying.* There is no need to create specific instructional settings. People spontaneously work in collaboration when the situation requires to do so, like in work and family settings.

7) In practice people may construct or re-construct their own r*ules-of-thumb* and informal *rules and laws* for managing actions, situations, materials and the environment in which they work. Such rules may be developed to manage specific situations like at work, e.g. rules for cleaning and storing tools after finishing a job. Therefor these rules are quite often *situation-based* and *situation- bound,* but may become more general when they are also applied in different situations.

In general, adult learning focuses on functional knowledge and practical skills that are straight applicable. Most learning is characterized by learning through doing and using authentic materials in communication with others. According to Greeno et al (1999) learning for participation in work and society can only take place in the workplace setting or in social communities in order to give meaning to the learning. Greeno states that learning in informal ways in the course of activity in a meaningful setting has shown to be much more effective than learning in classroom settings.

The above means that adults can also be their own teachers in many situations. They direct their own learning by processing information in their own ways. Except from learning by doing, new information in real life situations is quite often packed in sources like TV bulletins, internet, social media, newspapers and magazines, or comes in from colleagues, friends, neighbors, relatives and family. In particular internet and social media play an important role nowadays. Analyzing and understanding such information requires not only familiarity with IT means, but also literacy and numeracy skills. Ways in which adults process such information can be characterized by

- read about, listen to or look at information
- identify key points in the information
- communicate, discuss with others
- reflect on *what is "new" to me*?
- reflect on possible implications for personal life. *What does it mean to me?*
- reflect on possible implications for society or work.

When new information makes sense, then it will be integrated in already acquired knowledge and skills. (Van Groenestijn, 2002, Van Groenestijn & Lindenskov, 2007)

Teaching in adult education

Teaching adults has already been a topic for studies for many years. Freire (1970) posed teaching as a dialogue between learners and teachers: learning from experiences.

> *"The teacher is no longer merely the-one-who-teaches, but the one who is himself taught in dialogue with the students, who in turn while being taught also teach" (Freire, 1970, pg.67).*

After the IALS study Gal and Ginsburg (1996) introduced and discussed a number of instruction principles and strategies for teaching adults. These principles are included in appendix 1. Tout & Johnston (1995) developed a numeracy training course for teachers in Australia, named *Adult Numeracy Teaching - Making meaning in Mathematics*. Numeracy in the U.S. became part of the program *Equipped for the future*, that was launched in 1997 as a guide for lifelong learning.

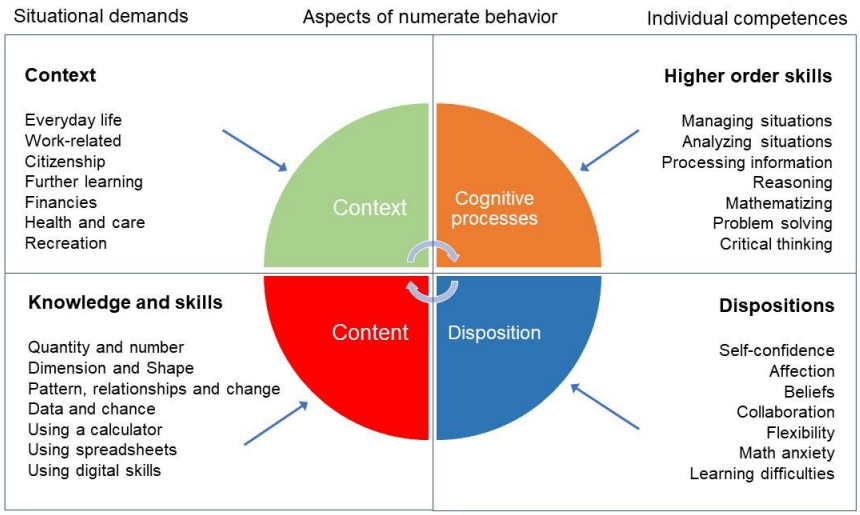
Gal (2000) edited the book *Adult Numeracy Development* in which adult learning and teaching was viewed from different perspectives on numeracy, approaches to instruction, practice and learning, and concluding with assessment of numeracy.
The five facets of numerate behavior, as described in the ALL and PIAAC surveys (Gal et al, 1999), offer also possibilities for the development of learning and teaching in adult education. Curry (2017) described each facet with content for learners in combination with instruction for teachers.
The complexity factors used to determine the level of numeracy items in both surveys, can be used as an indication for the levels of tasks in adult education (Gal et al, 2003, Tout et al, 2020). This was done in the MiA project by which the numeracy items for the participating countries could be validated. This was very useful for the teachers. The schema of the complexity factors is added in appendix 2. Remarks from teachers in MiA are in appendix 3.

The way in which teaching in adult education takes place may depend on the way courses are organized and the goals to achieve. There is a difference between formal, non-formal and informal courses. There is also a difference between group learning and distance learning. Though the common goal in all settings should be self-directed learning to prevent dependency from teachers. Adults are responsible for their own learning. This requires teachers to reflect on their ways of teaching in order to come to adaptive teaching.

Starting point in adult numeracy education is the wish of adults to learn what they need to learn and which is straight applicable in their daily lives. Adults may have built up a lot of experience in solving numeracy problems in lived-in situations in their own ways, sometimes even without being aware of using some math. They bring in prior knowledge and skills that may be based on a mix of formal and informal knowledge, plus their dispositions to math. This requires a way of adaptive teaching that fits their needs and ways of learning, but that also opens new ways for further learning and for better participation in society.

Teachers in adult education need first to be aware of the different roles adults have to fulfill and their dispositions for math. Second, they must respect the knowledge and skills their participants already have acquired in the course of life. Third, they must be familiar with math themselves and know the difference between mathematics as a subject and numeracy. These three components are the start for focusing on the perspectives of the individual participants.
In general, the focus in adult numeracy education is based on the four aspects of the CENF model for numerate behavior: the context, adults' dispositions concerning mathematics, the higher-order

processes that play a role when using mathematics and, last but not least, the content. Teachers must have developed competencies in these four areas for facilitating ways of learning which suit their learners best.



In each learning situation all four components play a role. Every learning situation starts from a problem in a *context* that needs to be *managed* or *solved* by using or doing *some math* but that may be influenced by the individual's *disposition* for math, with the perspective of improving this context or embedding the result in a new context, e.g. with the focus on a new job. Teachers must be aware of these components and try to pay attention to all four components in different ways during their sessions with participants, depending on the participants they work with.

Competencies teachers in adult education should have acquired are:
- being experts in numeracy themselves
- feel empathy for the learners
- have socio-political knowledge of the target groups they work with (minorities, women)
- create meaningful numeracy situations, preferable through practical tasks
- respect different ways of learning
- are able to listen to the learners' ways of reasoning and problem solving
- encourage critical thinking, communication, collaboration, creativity and flexibility
- are able to monitor and evaluate the learning process
- prevent dependency from learners of the teachers by encouraging autonomous learning.

These competencies are the ingredients for *adaptive teaching*. Professional teachers *don't teach*, but *facilitate learning* (Brookfield, 1986). They are able to finetune their teaching to their participants ways of learning by asking questions, encouraging communication, collaboration, critical thinking and by this *problem solving*. They support their learners to be their own teachers. Both, adult learners and their teachers, have their expectations during the numeracy sessions. Professional teachers are able to fulfill the expectations of their participants, but, at the same time, they will also try to get their expectations to a higher level.

The core of adult education is that teachers "enable learners to develop a collection of problem-solving strategies and the ability to apply them effectively, and mathematical reasoning so learners can make judgments about procedures and strategies based on understanding mathematical relationships." (Manly et al, 2006, pg. 36). Most important is that teachers, in contrary to school-like

sessions, create problem solving situations, preferable in out-school situations, in which adults are challenged to brainstorm, collaborate, reason and learn from each other. “With regards to affective factors, adult educators are challenged to support learners’ existing and developing sense of themselves as numeracy learners and doers, helping them to manage frustration with false starts during problem solving as well as to develop the confidence and agency to better manage real-life numeracy situations.” (Manly et al, 2006, pg. 36)

What content to teach

Following the international surveys described in the previous section, many OECD countries started numeracy courses as part of or in addition to already existing literacy courses. In 2006 Ginsburg, Manly and Schmitt published "The Components of Numeracy" for adult education. (Manly et al, 2006). In this publication they describe a methodology existing of three components: the context, the content, and the combined cognitive and affective component. The context of numeracy must be "real". The content is related to the ALL Survey (2002): Number and Operation Sense; Patterns, Functions and Algebra; Measurement and Shape; Data, Statistics, and Probability. The cognitive and affective component concerns conceptual understanding, adaptive reasoning, procedural fluency and productive disposition. All three components must be part of a curriculum for adult education. In the end, they discuss the curriculum and instruction, assessment and professional development.

In Australia the Australian Curriculum Assessment and Reporting Authority (ACARA, …….) published a numeracy curriculum for level 1 up to level 6 (spread over year 1 up to year 10). This curriculum describes content related to context. This curriculum is also usable for adult education. In the U.K. the *Adult Numeracy Core Curriculum* was published in 2001 by The Basic Skills Agency. Other European countries showed also more interest for numeracy in adult education. Numeracy programs were developed in Germany, The Netherlands, Norway, Spain? ……(more?)

In 2012 the 21st century skills for the future were published (Binkley et al, 2012). In total 12 competencies are described for becoming prepared for the future society. The four core competencies are critical thinking, communication, collaboration and creativity. Additional necessary competencies are information literacy, media literacy, technology literacy, flexibility, leadership, initiative, productivity and social skills. From the latter flexibility may be most important in its own to be able to adjust to and adapt new (technological) developments in society and work.

The OECD (2018) describes in "The Future of Education and Skills, *Education 2030"*, the need for being prepared for a complex and uncertain future. Students need to be prepared for *an increasingly volatile, uncertain, complex and ambiguous world in their future,* (OECD, 2018, pg. 4). Such requires a "*Need for a broad set of knowledge, skills, attitudes and values in action"* (OECD, 2018, p. 4).
"The OECD Learning Framework 2030 therefore encapsulates a complex concept: the mobilisation of knowledge, skills, attitudes and values through a process of reflection, anticipation and action, in order to develop the inter-related competencies needed to engage with the world."
(OECD, 2018, pg 7.)

The question is what this may mean for adults who are already at risk in literacy and numeracy whilst 2030 is nearing. This may require for them a double jump to keep up with the current developments in our technological and economic society and therefor also with societal changes. Together with the 21st century skills this enhances the need for qualitative numeracy (and literacy) education.
The 21st century skills and the 2030 framework lay the basis for lifelong learning in general, but in particular for literacy and numeracy in adult education. The main focus in both is on the higher order skills in order to be able to keep up with the ongoing developments in a globalized technological society. But at the same time this cannot been seen without the badly needed knowledge and skills, necessary to survive in the near future. This means that the content for numeracy education must be reviewed and restructured in combination with literacy and IT skills. The traditional content in mathematics, for example the paper based procedures as for the core operations addition, subtraction, multiplication and division, may not be important anymore in the future. The use of a

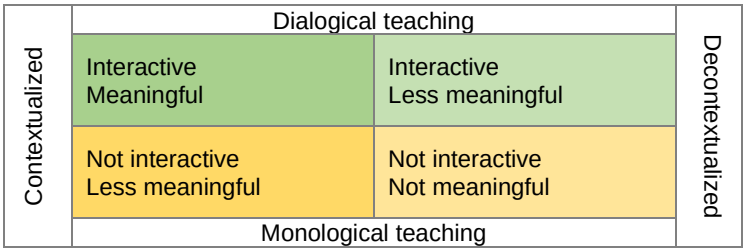
calculator and advanced programs as Excel and all kind of apps in combination with mental math are more important. Being familiar with numeracy related websites is also a must. But, at the same time understanding of the core operations and the concepts of fractions, decimals, percent, the metric system, time, some statistics, ratio and the relations between these concepts is highly important.

How to teach

In practice it appears that in particular the mathematical content highly influences the disposition of adults to mathematics, based on experiences in former compulsory education. Therefor the numeracy content must be meaningful in adult education. Starting from the CENF model of numerate behavior this means that the four components must be present in all numeracy courses:
- context based
- useful content
- enhancing higher order skills
- enabling positive, productive and constructive dispositions.

This also means that learning and teaching must go in an interactive way to help adults understand the usefulness of numbers in their own lived-in situations. Comparable to the literacy model for contextual/dialogical literacy teaching (Purcell et al, 1998) we may use a similar model for numeracy.

Figure….



This starting point has consequences for the way in which learning numeracy in adult education is organized. In an ideal situation learning mathematics is organized in authentic situations, for example on the work floor. In most situations, however, it is organized in centers for adult education. The challenging question for the CENF project is how to organize interactive numeracy activities in adult education in such a way that the gap between school and real life will be bridged.
Starting from the context, which is the lived-in situation of the participants, adults need to manage situations and solve problems that deal with numbers. By this, problem solving is the best way for learning and teaching in adult education. It challenges adults to activate their mathematical knowledge and skills and to critical thinking about the situation in which the problem is posed. It also activates dispositions that can be discussed. It challenges teachers to interactive meaningful and adaptive teaching. In the MiA project the following procedure and steps were used to practice problem-based learning and teaching (Van Groenestijn and Lindenskov, 2002). When it comes to analysis of numeracy problems in real-life situations the following details may be distinguished:

- There is a situation to manage or a problem to solve
- locate the situation or the problem as a mathematical situation, (it has something to do with numbers)
- identify the mathematics in it;
- analyze and structure the mathematical information in it
- interpret, give meaning to the mathematical information
- plan, discuss possible steps for solving the problem
- choose a solving procedure
- do computations, if needed, or act otherwise
- check the result

- apply contextual judgement, if necessary
- check possible consequences
- make decision
- reflect on the process.

To recognize the way adults learn in practical situations, the steps for acquiring and processing new information must be combined with the steps for managing numeracy situations and solving real life mathematical problems. The above means that learning processes in adult education should be organized in the best possible 'authentic' situation, an actual learning situation. This can be done through the following steps:

1) Bring the participant in a potential numeracy situation
Such a situation could be 'sales', for example. The teacher knows that they may encounter a mathematical problem in the situation. The teacher organizes a 'sales' situation by, for example:
- bringing the participants in an actual authentic situation, e.g. a department store or street market
- asking them to tell a story about their experiences concerning sales
- showing something in the classroom setting with a discount price, e.g. a coat priced 80 euro with a label: 15% off

2) Identify problems in the situation
Focus or zoom in on mathematical problems, e.g. a participant says: "I don't know how to compute the new price. I just pay the amount at the cashier desk they ask me to pay"
Activate prior knowledge: what do they actually know about percent?

3) Plan the problem solving procedure
The teacher challenges the learners to solve the problem: *"How do you think you can do this?"*
The participants may find all kinds of informal and formal problem solving procedures.
The teacher's task is to interact with the participants and try to discover what participants know and what they don't know, e.g. a participant states that 10% reduction is always 10 euro off.

4) Do the problem solving
At this point the learning process can start, e.g. by encouraging discussions among participants. Try to connect with their prior knowledge and good conceptions.
e.g. the participant knows that 50% is half. *How would you go on?*

5) Check the result
Can the participants explain why their answers or solutions are correct or not?

6) Reflect the process. What did the learners learn?
The participants discuss what they learned. *What is new to me? What does it mean for me in my personal life or in my work situation?*

These six steps can be applied in many actual real life situations. They challenge teachers to create an almost real-life situation in school settings. When teachers and participants are aware of these six steps then both, teachers and participants, are more and better actual involved in the learning process itself. It may help the participants to discover what they already know and what they really need and want to learn. It may help the teachers to find out how they can support and coach their participants in such a way that they feel independent and can organize their own learning processes. These six steps can also be of help for the teachers to organize their lessons in a structured way as well for the participants to learn to think in a structured way and by this learn to manage their lived-in situations.
Teachers need to be aware that learning should include the five core elements of the 21st century

skills, namely: critical thinking, communication, collaboration, creativity and flexibility. At the same time they are challenged to bring their participants to positive, productive and constructive dispositions by which they may also develop a good self-esteem.

Reminder for teachers:

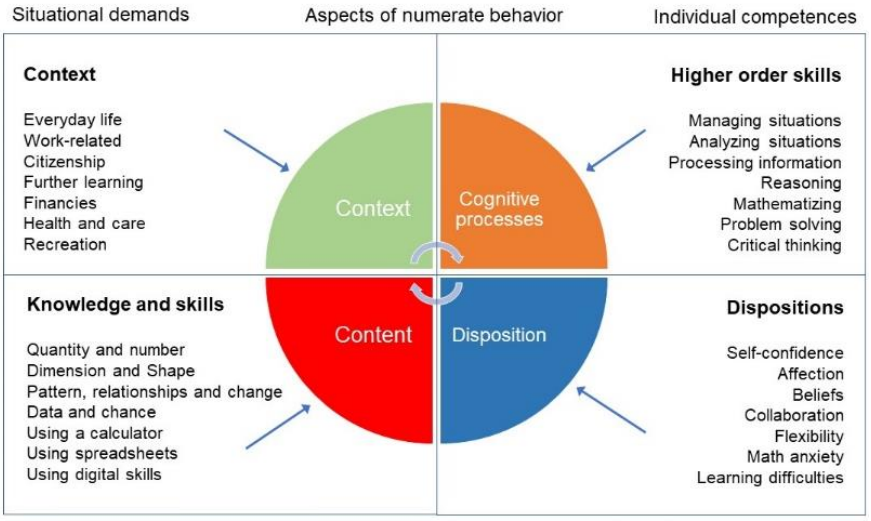
The best way of teaching adults is:
Don't start teaching
Start with posing a 'problem' in a real context
Ask questions to analyze and give meaning to the math in the problem
Challenge the participants to discuss, collaborate and solve the problem
Discuss the math in it
Evaluate the process.

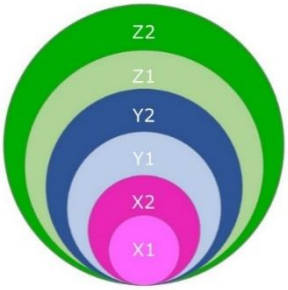
Setup of Modules for Professional Development

In order to come to a standard for qualitative adult education a set of modules has been developed for professional education of teachers and volunteers in adult education. These modules can be used in group work and for distance learning.
The goal of each module is threefold. The first goal is to enhance the teachers' awareness and competencies of qualitative and adaptive teaching. The second goal concerns the content and some relevant didactical principles, as for example how to 'teach' core computations, measurement, percent. The third goal is to improve the teachers' own numeracy skills in order to support them in their self-esteem in teaching numeracy.

The four components as described in the CENF graph for numerate behavior, are the core of these modules. In each module at least three out of four components will be present: context, content, disposition and/or higher order skills.

The way in which each module has been setup is based on background information and the six steps of problem solving. The focus is on adults at levels X1, X2 and Y1 as described in the CENF background information. (IO3)





Each module consists of elements of the following parts:

Individual's competencies	Mathematical content knowledge and skills	Tools and skills Tools

Context	Understanding and using: whole numbers, fractions, decimals metric system, 2D, 3D ratio, percent some statistics pattern and chance	Paper based Calculator Excel ICT PPT Video Literature
Disposition		
Higher order skills		

Points for attention and competencies to acquire:

- The mathematical content is, in fact, not only the goal to achieve but also the means to overcome possible problems with context and dispositions by way of focusing on higher order skills. For this, teachers need to be aware of questions their participants may have but don't know how or don't dare to pose. This means that teachers must be aware of body-language.
- Teachers will be trained in the six steps of problem solving themselves.
- Teachers will be trained in adaptive and dialogical teaching.
- In addition, the teachers will be challenged to improve their own mathematical skills in relation to numeracy skills.(CENF Y2 , Z1?)

Each module starts from a context
In each module some aspects of disposition and/or higher order skills must be present.
In each module some mathematical content related to the context will be discussed.

Thereby, the focus in each module will be on:

- The teachers themselves, keeping in mind how to transfer their experiences in meetings with their participants
- becoming aware of needed higher order skills to manage the context situation
  improving some disposition, if needed.
- Improving numeracy / mathematical skills,

The question to think about is still assessment of teachers for certification.

The modules will be split in mini-modules where as the first half will be spent on the individual's competencies and the second part on content and tools.

Example setup of PDM module: Sales, level Y1

Context Sales and Dispositions: collaboration	Subject sales: discussion about experiences with sales. How do teachers themselves deal with sales and percent? How about their participants? Subject collaboration: work in duos Suggestions how to work in duos. How do Teachers think about percent. What could be difficult for their participants? How to teach percent?
Content: percent off some products with different offers: T-shirts: half price A coat: 20% off A television: 15% off	Tools: Work with easy percentages off (50, 25, 10, 5, 20%) Ways of doing computations with percent Using a table (paper based) Using a calculator

Teacher Trainer (TT) and Teachers (T)

Activities	**1** Organize **collaboration** in a group session; work in duo's; exchange in different duo's; discuss the process: what did they learn from each other? **2** Ask the teachers how they would do 10% off themselves. Discuss doing computations with **percentages**	1 competence 2 content
Step 1 Bring the teachers in a numeracy situation	Context – sales coats 10% off What is the offer? What is it about? What does percent mean? What is the question?	
Step 2 Analyze the situation	Analyze the math in it. What do the teachers know about %? Have them explain about %	What math skills are necessary to do computations with percent?
Step 3 activate prior knowledge	How do teachers deal with the question? Have them tell. How did you learn about percent? Have them explain about 100% - 50% -25% -10%	Have them reason How does % work with a calculator?
Step 4 solve the problem	Have the teachers do the computations in their own ways. Discuss different ways of doing computations with percent. Ask them to work in duos; change duos	Use a table Use a calculator
Step 5 What is the conclusion?	What is the new price? How did they find the answer? What happens if the offer would be 20% off? Or 25% off? or 15% off? Use coats with different prices.	Answer?
Step 6 Evaluate the process with the learners	Check the conclusion. Have the teachers discuss about percent. How do they reason?	
Evaluation T	Evaluate the process. How did the teachers collaborate? Did they learn from each other? What did they learn? Did I teach? What did I teach? How to teach participants? Discuss the way of adaptive teaching.	
Next Activity	??..... percent extra / VAT	

Sales: coat 80 euro; 10% off
What is the new price during Sales?
Use a table. What happens when 15% off?

100%	80
50%	
25%	
10%	
5%	
15%	
20%	
1%	

Using a calculator: compare the results with the table on the left side.

80 – 50%
80 – 25%
80 – 10%
80 – 5%
80 – 15%
80 – 20%

80 x 50%
80 x 25%
etc.

What happens with 80 + 50% ? (explain)
How to compute VAT?

Change the price of the coat to 120,00
sales: 15% off

0,5 %		

Transfer to Teacher (T) and Participants (P) – Teachers practice with their participants

Context	Everyday life	Points of attention
Numeracy situation	Shopping Sales 10% off	
Disposition T	Like or dislike %; What do I know about percentages?	
Disposition P	Not familiar with % ? didn't like math in school?	
Competence P	Learn to collaborate; learn about %	
Competence T	Discuss different ways of **collaboration** Discuss different ways of doing computations with **percentages** (10% off)	
Activities T Organization of the session	**1a** Organize collaboration in a group session with participants; work in duos; **2** Ask the participants how they would do 10% off discuss doing computations with percent **1b** change to different duos; discuss the different ways of doing computations with percent: **1c** evaluate the process: what did they learn from each other?	
Step 1 Bring the P in a numeracy situation	Content – sales coats 10% off What is the offer? What is it about? Give meaning to it. What is the question?	
Step 2 Analyze the situation	Analyze the math in it. What do the participants know about %? Have them explain about %	
Step 3 activate prior knowledge	How do learners deal with the question? Have them tell. What is new for them? What do they need to learn? Have them tell. Have them explain about 100% - 50% -25% -10%	Have them reason How does % work with a calculator?
Step 4 solve the problem	Have them do the computations in their own ways Ask them to work in duos change duos: discuss the findings	Use a table Use calculator
Step 5 What is the conclusion?	What is the new price? How did they find the answer? What happens if the offer would be 20% off? Or 25% off?	Answer?
Step 6 Evaluate the process with the learners	Check the conclusion. Have them discuss. How do they reason?	
Evaluation T	Evaluate the process. How did the learners collaborate? Did they learn from each other? What did they learn? Did I teach? What did I teach?	
Next Activity	??..... percent extra; VAT	

In this way the prior knowledge of participants is recognized. The teacher's task is to coordinate the discussion to enable all participants to share their feelings for percent and how they deal with percent in their own ways. Together with the teacher some efficient ways of doing computations with percent may be practiced by using the table and a calculator.

Early list of modules - background information

1. Mastering numeracy as a "user" and meta knowledge of the subject
2. Socio-political orientation knowledge
3. Target group knowledge, andragogical know-how and know-how about the specific learning processes in numeracy
4. Roles and skills as an adult educator with a focus on enabling didactics (empowerment)
5. Theories of education and learning (autonomous learning, situational learning, connectivity, ...)
6. Competence-oriented teaching and learning advice
7. Development and planning of learning opportunities, methods and materials (didactization of numeracy)
8. Media literacy and digital basic education in the MINT subject
9. Heterogenity, internal differentiation, individualization
10. Communication, cooperation, collaboration
11. Gender and diversity
12. Life work balance

(to be discussed)

Malcolm (1996, 2002) describes adult learning as a two-way process.
Malcolm, (2002, p 95)

Teaching concepts can be divided into two main groups:
- those which focus mainly on the perspective of the organization providing education or training;
- those which are more concerned with the perspective of the individual learner, namely content and conscientization.

The first group concerns the organization of traditional group learning, but also learning on distance, flexible learning and open learning. The second group includes experiential, problem-based, independent and self-directed learning.
The latter group can further be distinguished by learning focusing on the mathematical content and on conscientization. With conscientization is meant the process of becoming a critical thinker concerning societal, economic and cultural influences and developments.

Assessment

The PD-training will be rounded off with assessment and a European based certificate (EQF level 7). (….)

Criteria:

The teacher

- has mastered at least numeracy level Y2
- shows to have knowledge about target groups
- shows to be able to create learning situations in which adults feel free *to do* math activities in their own ways and in collaboration with others
- is familiar with adults' dispositions that may influence their feelings and attitude concerning numeracy
- shows to be able to have adult learners communicate, collaborate, to do problem solving and to reflect on their actions.
- is able to have the participants work with calculator and IT-programs.

Conclusion

It is strongly advised to create learning situations in which adults feel free *to do* math activities in collaboration with others. Such situations must be real-life related. Adults must feel free and challenged to do their own ways of problem solving: analyzing the situation, brainstorming what to do, finding an effective solution, challenging to get at a higher level of numeracy, reflecting on their own learning process.

The underlying processes based on this way of learning are: developing self-esteem, building well-understood mathematical concepts and procedures, managing numeracy-related situations.

CENF offers a set of professional development modules (PDM) for teachers in adult education, keeping in mind the above described core competencies for adult learners.
Learning numeracy must be meaningful.

The modules focus on three aspects: adult learning, teaching adults and numeracy.
The key target of CENF is to work on international enhancement of numeracy in the European countries. Such means that CENF must also provide ways that enable countries to organize adult learning settings for becoming more numerate, if possible also in combination with adult literacy courses.

That raises a few questions:
- who are the learners? Which adults are most vulnerable concerning numeracy?
- who are the teachers?
- how to setup numeracy activities or courses for vulnerable adults in society?

It must be evident that the most stress is on self-management and functional numeracy, enabling adults to manage their own real-life situations.

The PDM-s will exist of two sets of mini-modules:

- modules that focus on higher order skills and dispositions
- modules that focus on didactical issues for teaching numeracy.

Starting point is always the lived-in situation from the participants.

Appendix 1

After the IALS study Gal and Ginsburg (1996) introduced and discussed a number of instruction principles and strategies for teaching adults.

1 Address and evaluate attitudes and beliefs regarding both learning math and using math.
2 Determine what students already know about a topic before instruction.
3 Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations or hands-on materials.
4 Encourage the development and practice of estimation skills.
5 Emphasize the use of "mental math" as a legitimate alternative computational strategy and encourage development of mental math skill by making connections between different mathematical procedures and concepts.
6 View computation as a tool for problem solving, not an end in itself.
7 Encourage use of multiple solution strategies.
8 Develop students' calculator skills and foster familiarity with computer technology.
9 Provide opportunities for group work.
10 Link numeracy and literacy instruction by providing opportunities for students to communicate about mathematical issues.
11 Situate problem-solving tasks within meaningful realistic contexts in order to facilitate transfer of learning.
12 Develop students' skills in interpreting numerical information appearing within documents and text.
13 Assess a broad range of skills, reasoning processes and dispositions, using a range of methods.

Summarizing the core of these principles:
- start from prior knowledge,
- provide opportunities for group work
- focus on the usefulness of math in real life situations,
- view computations as a tool for problem solving, not as an end in itself,
- encourage adults to communicate about mathematical issues
- encourage problem solving strategies in meaningful realistic contexts
- encourage the use of a calculator in combination with estimation and mental math
- encourage becoming familiar with computer technology.

Nowadays these principles are still valid.

Gal and Ginsburg (2006). Instructional strategies for teaching adult numeracy skills. Philadelphia, NCAL technical report TR96-02, May 1996.

Appendix 2

Complexity factors for analyzing the difficulty of numeracy items

Tout, D., Gal, I., van Groenestijn, M., Manly, M., & Schmitt, M. J., (2020). *PIAAC Numeracy Task Complexity Schema: Factors that impact on item difficulty*. Australian Council for Educational Research. https://doi.org/10.37517/978-1-74286-609-3.

Aspects	Category	Range
Textual aspects	1. Type of match/problem transparency	Obvious/explicit to embedded/hidden
	2. Plausibility of distractors	No distractors to several distractors
Mathematical aspects	3. Complexity of Mathematical information/data	Concrete/simple to abstract/complex
	4. Type of operation/skill	Simple to complex
	5. Expected number of operations/processes	One to many

Table 1: Complexity Factors—Overview

Appendix 3

MiA teachers' views on good practices (MiA handbook, pp. 38-39)

To MiA teachers good practice means:

Teachers facilitate and coach adult learners' learning.

Teachers focus on how to best stimulate the learners, how to best ask questions and how to best summarise learners' work.

Teachers are aware of and reflect upon how they best support that adult learners come up with own examples, tell stories and explain material.

Teachers are aware of and reflect upon how they best pose their questions to the adult learners.

Teachers are aware of and reflect upon how they best support adult learners to report on their findings. Good practice in MiA is that teachers are aware and reflect upon how they best react when adult learners working either individually or in groups come to a wrong solution.

The attitude of teachers has to be respectful towards the learners. They listen to the learners and give them the opportunity to experience that they can do more and know more than they often think they can.

Teachers can pose questions like 'Where do you think you need maths in your daily life?' and 'Where do you experience that some mathematics is a problem for you?', 'Where do you need mathematics to solve a problem?', 'Where is it not a problem for you?', and 'Where does mathematics help you?'.

Teachers can ask the learners about situations they have experienced, what they think they understand and do not understand, and what they would like to know and be able to do.

Teachers can ask learners to try to work on some of their own problems, let them themselves summarize what they have done, and let them see how differently they solved it. Teachers can ask the learners to try to do it in another way than they usually do, and then let them explain to each other how you did. Teachers can ask learners to slightly change the context or the figures, and let them solve it again and explain what they found out.

Teachers can support learners by having them tell each other how they do and how to do computations in specific situations. That does not mean that teachers can never explain anything. Teachers can support learners to develop formal ways of doing computations or by showing formal ways to the learners and talk about how they relate to learners methods.

Teachers should not say 'you must do it this way!'

Teachers can ask learners to describe a situation and ask if they see any mathematics in the situation. When some learners know a lot, others know a little or nothing, then the learners can discuss among themselves how they will go on, they can choose one of the suggestions and try out as a start, and everybody has to come to a kind of understanding. Teachers can best support by e.g. suggesting how to go on, if the group is stuck, by saying some positive things as 'please go on', and if they know that they are not on the right track, they could suggest that they find the consequences. Then the

learners maybe will go back and choose a different way, and if they don't understand the teacher may give some small hints.

The teacher ensures that the learners reflect on what they have been doing. He/ she can go through the process step by step in a dialogue with the group. The teacher assures that everybody participates.

The teacher supports the learners to try out similar problems in a different context or with other figures and measures.

Good practice means that the teacher invites the learners to participate in a try to generalize their findings.

Teachers are aware of and reflect upon how to best involve the learners in deciding and presenting a situation to work on. Learners may think that they are not allowed to come up with their own situations and problems. One idea is to ask the learners to tell about what they did the day before. The teacher could start telling about his own experiences the day before as inspiration. Another idea is to stimulate the learners to make an investigation about what can be mathematics and mathematical problems in situations in their life. And finally, let the learners tell stories and then the teacher suggests in what way mathematics come into play. Such examples can stimulate the learners to find examples themselves.

The teacher analyses situations together with the students to formulate what actually is the problem in the situation, which information do you need, what is relevant to know about, what is not relevant to know about, which things in the situation do you understand and not understand, and do you think it looks like something you know from other situations? From the situation analysis specific problems can be formulated.

Teachers ask the learners to find different ways of solutions and assure that all learners can see possible ways of solutions so everyone can choose what they prefer. Then teachers or learners present a slightly different problem to be solved with the method the learners prefer.

Teachers invite the learners to wonder about which consequences can be taken by persons or society according to the results of the problem solving.

It is not necessary that the teacher always knows the answers. He/she can find the topic interesting, can participate in discussions and is curious to learn.

Teachers realize that the same situations do not interest all people. When you win an amount of money, you are interested in how to use the money and how to save and invest. When in a shop you can calculate 50% and 25% discount, but need to calculate 20% without being able to do it, then you are interested to learn to calculate 20%. When you wish to make scarves as Christmas gifts and have some fabrics 1.2 meter times 6 meter, you are interested to know what kinds of scarves are possible to make.

PDM2- Critical thinking (voorbeeld)

Example setup of PDM module: Sales, level Y1

Context Sales and Dispositions: collaboration	Subject sales: discussion about experiences with sales. How do teachers themselves deal with sales and percent? How about their participants? Subject collaboration: work in duos Suggestions how to work in duos. How do Teachers think about percent. What could be difficult for their participants? How to teach percent?
Content: percent off some products with different sales: T-shirts: half price A coat: 20% off A television: 15% off	Tools: Work with easy percentages off (50, 25, 10, 5, 20%) Ways of doing computations with percent Using a table (paper based) Using a calculator

Nederland heeft een inwonertal van 17.440.679 (1 september 2020) en met een oppervlakte van 41.543 km² een hoge bevolkingsdichtheid van bijna 420 inwoners/km². Daarmee is Nederland het op vijftien na dichtst bevolkte land van de wereld; met de kleine stadstaten Monaco , Vaticaanstad en San Marino niet meegerekend is Nederland na Malta het dichtstbevolkte land in Europa.

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